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1. Diagonals $\overline{AC}$ and $\overline{BD}$ of convex quadrilateral $ABCD$ meet at $P$. Prove that the incenters of the triangles $\triangle PAB$, $\triangle PBC$, $\triangle PCD$, $\triangle PDA$ are concyclic if and only if their $P$-excenters are also concyclic.

2. Let $n > 1$ be a positive integer. Each cell of an $n \times n$ table contains an integer which is $1 \pmod{n}$. It is given that the sum of the numbers in any row, as well as the sum of numbers in any column, is congruent to $n \pmod{n^2}$. Denote by $R_i$ the product of the numbers in the $i^{\text{th}}$ row and by $C_j$ the product of the numbers in the $j^{\text{th}}$ column. Prove that

$$R_1 + \cdots + R_n \equiv C_1 + \cdots + C_n \pmod{n^4}.$$

3. Let $n \geq 3$ be a fixed integer. An $n \times n$ table of nonzero real numbers is given with the property that for any cell, the sum of the $2n - 2$ numbers in the other cells of the same row or column is exactly $k$ times the number in that cell. Determine all possible values of $k$, in terms of $n$.

4. Let $ABC$ be a triangle with $AB = AC$, and let $M$ be the midpoint of $\overline{BC}$. Let $P$ be a point such that $PB < PC$ and $\overline{PA} \parallel \overline{BC}$. Let $X$ and $Y$ be points on lines $PB$ and $PC$ such that $B$ lies inside segment $PX$, $C$ lies inside segment $PY$, and $\angle PXM = \angle PYM$. Prove that quadrilateral $APXY$ is cyclic.

5. Let $n \geq 2$ be an integer. There are $n^2$ marbles, each of which is one of $n$ colors, but not necessarily $n$ of each color. Prove that they may be placed in $n$ boxes with $n$ marbles in each box, such that each box contains marbles of at most two colors.

6. A point $T$ is chosen inside a triangle $ABC$. Let $A_1$, $B_1$, $C_1$ be the reflections of $T$ in $BC$, $CA$, and $AB$, respectively. Let $\Omega$ be the circumcircle of $\triangle A_1B_1C_1$. The lines $A_1T$, $B_1T$, $C_1T$ meet $\Omega$ again at $A_2$, $B_2$, $C_2$, respectively. Prove that lines $AA_2$, $BB_2$, $CC_2$ meet on $\Omega$.

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1. Determine all pairs $(m, n)$ of positive integers for which there exists a positive integer $s$ such that $sm$ and $sn$ have an equal number of divisors.

2. A point $T$ is chosen inside a triangle $ABC$. Let $A_1$, $B_1$, $C_1$ be the reflections of $T$ in $BC$, $CA$, and $AB$, respectively. Let $\Omega$ be the circumcircle of $\triangle A_1B_1C_1$. The lines $A_1T$, $B_1T$, $C_1T$ meet $\Omega$ again at $A_2$, $B_2$, $C_2$, respectively. Prove that lines $AA_2$, $BB_2$, $CC_2$ meet on $\Omega$.

3. Let $n \geq 3$ be an integer. Prove that there exists a set $S$ of $2n$ distinct positive integers such that for any $m \in \{2, \ldots, n\}$, the set $S$ can be partitioned into two sets with cardinalities $m$ and $2n - m$ with equal sums.

4. Determine whether there exists a sequence $a_1, a_2, \ldots$ of nonnegative reals such that
$$a_n + a_{2n} + \cdots = \frac{1}{n}$$
for every positive integer $n$.

5. Determine all functions $f\colon \mathbb{R}_{>0} \to \mathbb{R}$ satisfying
$$\left(x + \frac{1}{x}\right) f(y) = f(xy) + f\left(\frac{y}{x}\right)$$
for all $x, y > 0$.

6. For which positive integers $n$ does there exist an $n \times n$ matrix with entries in $\{-1, 0, 1\}$ such that all $2n$ row sums and column sums are pairwise distinct?

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5. Determine all functions $f \colon \mathbb{R}_{>0} \to \mathbb{R}$ satisfying
$$\left(x + \frac{1}{x}\right) f(y) = f(xy) + f\left(\frac{y}{x}\right)$$
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## #?????? $\rightarrow$ $\boxed{*}$****

1. The numbers $1, 2, \ldots, 1024$ are written on a blackboard. The following procedure is performed ten times: partition the numbers on the board into disjoint pairs, and replace each pair with its nonnegative difference. Determine all possible values of the final number.

2. Let $n \geq 3$ be a fixed integer. An $n \times n$ table of nonzero real numbers is given with the property that for any cell, the sum of the $2n-2$ numbers in the other cells of the same row or column is exactly $k$ times the number in that cell. Determine all possible values of $k$, in terms of $n$.

3. Let $\mathbb{Q}_{>0}$ denote the positive rational numbers. Find all functions $f\colon \mathbb{Q}_{>0} \to \mathbb{Q}_{>0}$ obeying
$$f\left(x^2 f(y)^2\right) = f(x)^2 f(y)$$
for all positive rational numbers $x$ and $y$.

4. Let $ABC$ be a triangle with $AB = AC$, and let $M$ be the midpoint of $\overline{BC}$. Let $P$ be a point such that $PB < PC$ and $\overline{PA} \parallel \overline{BC}$. Let $X$ and $Y$ be points on lines $PB$ and $PC$ such that $B$ lies inside segment $PX$, $C$ lies inside segment $PY$, and $\angle PXM = \angle PYM$. Prove that quadrilateral $APXY$ is cyclic.

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## #?????? $\rightarrow$ $\boxed{*}$* *******

1. A circle $\omega$ of radius 1 is given. A collection $T$ of triangles is *good* if

   - each triangle in $T$ is inscribed in $\omega$;
   - no two triangles in $T$ share a common interior point.

   Determine all $t > 0$ such that, for any $n$, there exists a good collection of $n$ triangles, all with perimeter greater than $t$.

2. Let $\mathbb{Q}_{>0}$ denote the positive rational numbers. Find all functions $f\colon \mathbb{Q}_{>0} \to \mathbb{Q}_{>0}$ obeying
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for all $x, y > 0$.

6. Let $a_0$, $a_1$, $a_2$, $\ldots$, $a_{2018}$ be a sequence of real numbers such that $a_0 = 0$, $a_1 = 1$, and for every $n \geq 2$ there exists $1 \leq k \leq n$ satisfying
$$a_n = \frac{a_{n-1} + \cdots + a_{n-k}}{k}.$$
Find the smallest and largest possible values of $a_{2018} - a_{2017}$.

## #?????? $\to$ $\boxed{*}$******* ***********

1. Let $n$ be a positive integer. Determine the number of sequences of real numbers $a_1, \ldots, a_n$ such that
$$a_{k+1} = a_k^2 + a_k - 1$$
for all $k$ (with indices modulo $n$).

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